ΥΠΟΥΡΓΕΙΟ ΠΑΙΔΕΙΑΣ ΚΑΙ ΠΟΛΙΤΙΣΜΟΥ
ΔΙΕΥΘΥΝΣΗ ΑΝΩΤΕΡΗΣ ΚΑΙ ΑΝΩΤΑΤΗΣ ΕΚΠΑΙΔΕΥΣΗΣ
ΥΠΗΡΕΣΙΑ ΕΞΕΤΑΣΕΩΝ

ΠΑΓΚΥΠΡΙΕΣ ΕΞΕΤΑΣΕΙΣ 2006

Μάθημα: ΜΑΘΗΜΑΤΙΚΑ
Ημερομηνία και ώρα εξέτασης: Δευτέρα, 29 Μαΐου 2006
7.30 π.μ. - 10.30 π.μ.

ΛΥΣΕΙΣ

ΜΕΡΟΣ Α

1.	$\int(3x-\eta\mu x)dx=\frac{3x^2}{2}+\sigma\upsilon\nu x+c$	
2.	$\frac{x^2}{9}+\frac{y^2}{4}=1$, $\frac{x^2}{\alpha^2}+\frac{y^2}{\beta^2}=1$ $\Leftrightarrow\ \alpha^2=9\ \Leftrightarrow\ \alpha=3$ $\beta^2=4\ \Leftrightarrow\ \beta=2$ $\gamma=\sqrt{\alpha^2-\beta^2}=\sqrt{9-4}=\sqrt{5}$ Κορυφές : $A(3\,,0)$ $A'(-3\,,0)$ $B(0\,,2)$ $B'(0\,,-2)$ Εστίες : $E(\gamma\,,0)$ $E'(-\gamma\,,0)$ $E(\sqrt{5}\,,0)$ $E'(-\sqrt{5}\,,0)$ Εκκεντρότητα : $\varepsilon=\frac{\gamma}{\alpha}\ \Rightarrow\ \varepsilon=\frac{\sqrt{5}}{3}$	
3.	$y=6x^2$ $E=\int_1^2 6x^2dx$ $=\left[\frac{6x^3}{3}\right]_1^2=\left[2x^3\right]_1^2$ $=2\cdot8-2\cdot1$ $=14$ τ.μ.	

4.

$$\lim_{x\to 0}\frac{x+\ln(x+1)}{e^x-1}=\frac{\ln 1}{e^0-1}=\frac{0}{1-1}=\frac{0}{0}\quad \text{Απροσδ.}$$

$$\overset{\text{DLH}}{=}\lim_{x\to 0}\frac{1+\frac{1}{x+1}}{e^x}=\frac{1+\frac{1}{0+1}}{e^0}=\frac{1+1}{1}=2$$

5.

Ε	Δ	Μ
6	5	4

$\Rightarrow 6\cdot 5\cdot 4=120$ τριψήφιοι

6.

$$\left.\begin{aligned} x_\kappa&=\frac{1+3}{2}=2\\ y_\kappa&=\frac{1+5}{2}=3\end{aligned}\right\}\Rightarrow K(2,3)$$

$$R=\sqrt{(3-1)^2+(2-1)^2}$$

$$=\sqrt{4+1}=\sqrt{5}$$

Εξίσωση κύκλου : $(x-2)^2+(y-3)^2=5$

7.

$$A=\begin{pmatrix}3 & -1\\ -5 & 2\end{pmatrix}\quad,\quad B=\begin{pmatrix}2 & 4\\ 1 & 5\end{pmatrix}$$

(α) $\Gamma=A\cdot B=\begin{pmatrix}3 & -1\\ -5 & 2\end{pmatrix}\cdot\begin{pmatrix}2 & 4\\ 1 & 5\end{pmatrix}$

$$\Gamma=\begin{pmatrix}5 & 7\\ -8 & -10\end{pmatrix}$$

(β) $B^{-1}=\frac{1}{|B|}\cdot\begin{pmatrix}5 & -4\\ -1 & 2\end{pmatrix}$

$$B^{-1}=\frac{1}{6}\cdot\begin{pmatrix}5 & -4\\ -1 & 2\end{pmatrix}$$

8.	(α) $P(B)=1-P(B')$ $=1-\frac{1}{4}$ $=\frac{3}{4}$ (β) $P(A\cap B)=P(A)+P(B)-P(A\cup B)$ $=\frac{1}{3}+\frac{1}{4}-\frac{53}{60}$ $=\frac{20+45-53}{60}$ $=\frac{12}{60}=\frac{1}{5}$ (γ) $P(A-B)=P(A)-P(A\cap B)$ $=\frac{1}{3}-\frac{1}{5}$ $=\frac{5-3}{15}=\frac{2}{15}$	
9.	$y^2=4\alpha x$ $2y\frac{dy}{dx}=4\alpha$ $\Rightarrow \frac{dy}{dx}=\frac{2\alpha}{y}$ $\Rightarrow \lambda_{\varepsilon\varphi}=\left.\frac{dy}{dx}\right\|_{y=y_1}=\frac{2\alpha}{y_1}$ Εφαπτόμενη στο $A(x_1\ ,\ y_1)$: $y-y_1=\lambda_{\varepsilon\varphi}(x-x_1)$ $y-y_1=\frac{2\alpha}{y_1}(x-x_1)$ $y_1y-y_1^2=2\alpha x-2\alpha x_1$, $y_1^2=4\alpha x_1$ $y_1y=2\alpha x+2\alpha x_1$ $y_1y=2\alpha(x+x_1)$	
10.	$\int\frac{x^3}{(x^2+1)^2}dx=$ $x^2+1=u$ $2x\,dx=du$ $x^2=u-1$ $=\int\frac{x^2\cdot x\cdot dx}{(x^2+1)^2}$	

$$= \frac{1}{2}\int \frac{(u-1)}{u^2}\,du$$

$$= \frac{1}{2}\left[\int \frac{u}{u^2}\,du - \int \frac{1}{u^2}\,du\right]$$

$$= \frac{1}{2}\left[\int \frac{1}{u}\,du - \int \frac{1}{u^2}\,du\right]$$

$$= \frac{1}{2}\left[\ln u + \frac{1}{u}\right] + c$$

$$= \frac{1}{2}\left[\ln\left(x^2+1\right) + \frac{1}{x^2+1}\right] + c$$

ΜΕΡΟΣ Β

1.

$$y = \frac{x^2+1}{x^2-4}$$

Π.Ο. $x \in R - \{-2\,,\,2\}$ $\Rightarrow$ $x = 2\,,\; x = -2$ Κ.Α. διότι μηδενίζεται μόνο ο παρονομαστής.

Σημεία Τομής: $x = 0 \Rightarrow y = -\frac{1}{4} \Rightarrow \left(0\,,\,-\frac{1}{4}\right)$

$y = 0 \Rightarrow x^2 + 1 = 0 \Rightarrow$ ρίζες μιγαδικές

$\Rightarrow$ δεν υπάρχουν τομές με άξονα x.

Ακρότατα $y' = \dfrac{2x\left(x^2-4\right) - 2x\left(x^2+1\right)}{\left(x^2-4\right)^2} = \dfrac{2x^3 - 8x - 2x^3 - 2x}{\left(x^2-4\right)^2} = \dfrac{-10x}{\left(x^2-4\right)^2}$

$y' = 0$

$\Rightarrow -10x = 0 \Rightarrow x = 0$

x	$-\infty$	-2		0		2	$+\infty$
y'	+	‖	+	0	−	‖	−
y	↗	‖	↗	⌒	↘	‖	↘
				$\max\left(0\,,\,-\frac{1}{4}\right)$			

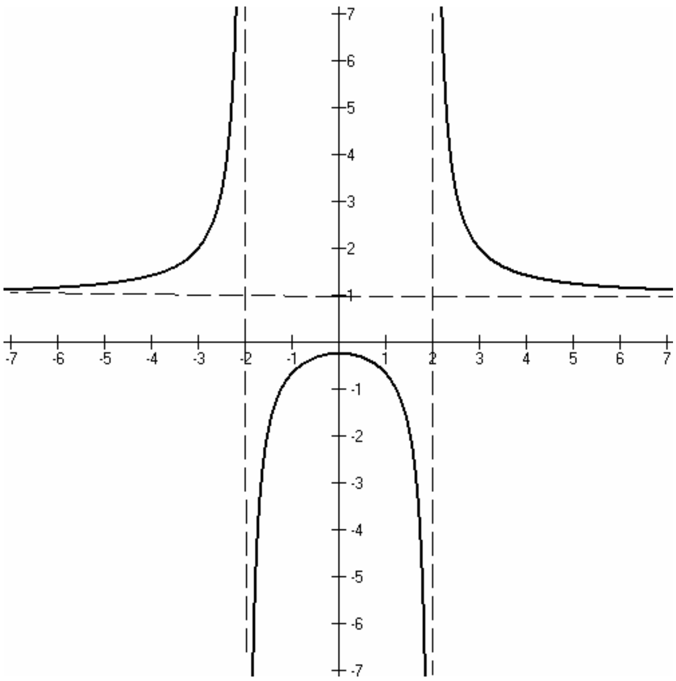
Για x = 0 $\Rightarrow y_{max} = -\frac{1}{4}$

Ασύμπτωτες: $\lim_{x \to \pm\infty} \frac{x^2+1}{x^2-4} = \frac{\infty}{\infty}$ απροσδ.

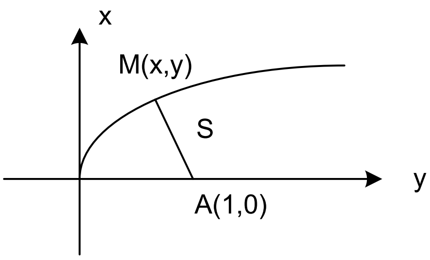
D.L.H.

$= \lim_{x \to \pm\infty} \frac{2x}{2x} = 1 \quad \Rightarrow y = 1$ Ο.Α.

και $\Rightarrow x = 2\ ,\ x = -2$ Κ.Α.



2. $y=\sqrt{x}\ ,\quad x>0\ ,\quad A(1\,,\,0)$



$$S=(AM)=\sqrt{(x-1)^2+(y-0)^2}=\sqrt{x^2-2x+1+y^2}$$

$$=\sqrt{x^2-2x+1+x}=\sqrt{x^2-x+1}$$

$$\frac{dS}{dx}=\frac{2x-1}{2\sqrt{x^2-x+1}}=0$$

$$2x-1=0 \quad \Leftrightarrow \quad x=\frac{1}{2}$$

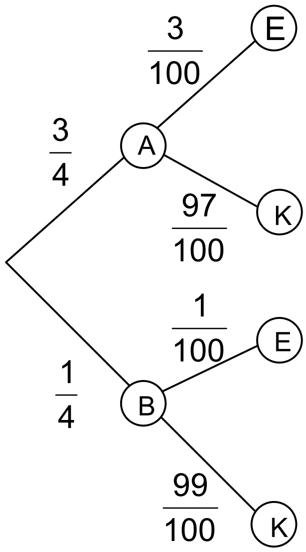
x	0^+		$\frac{1}{2}$		$+\infty$
$\frac{dS}{dx}$		-	0	+	
S		↘	min	↗	

Για $\quad x=\frac{1}{2} \qquad \Rightarrow y_{min}=\sqrt{\frac{1}{2}}=\frac{1}{\sqrt{2}}=\frac{\sqrt{2}}{2}$

$$\Rightarrow\ M\left(\frac{1}{2},\frac{\sqrt{2}}{2}\right)$$

$$\Rightarrow S=(AM)=\sqrt{\left(\frac{1}{2}\right)^2-\frac{1}{2}+1}=\sqrt{\frac{1}{4}-\frac{1}{2}+1}=\sqrt{\frac{3}{4}}=\frac{\sqrt{3}}{2}$$

3. δεντροδιάγραμμα



(α) $P(A)=\dfrac{N(A)}{N(\Omega)}=\dfrac{15000}{20000}=\dfrac{3}{4}$

$P(B)=\dfrac{1}{4}$

(β) $P(E)=P(E\cap A)+P(E\cap B)$

$=P(E/A)\cdot P(A)+P(E/B)\cdot P(B)$

$=\dfrac{3}{100}\cdot\dfrac{3}{4}+\dfrac{1}{100}\cdot\dfrac{1}{4}$

$=\dfrac{9}{400}+\dfrac{1}{400}=\dfrac{10}{400}=\dfrac{1}{40}$

(γ) $P(A/E)=\dfrac{P(A\cap E)}{P(E)}$

$=\dfrac{\frac{9}{400}}{\frac{1}{40}}=\dfrac{9}{10}$

4.

(α) $I=\int_0^{\pi} f(x)g(x)dx$, $x = \pi - y$

χ	0	π
y	π	0

$dx = -dy$

$$=\int_{\pi}^{0} f(\pi-y)g(\pi-y)(-dy)$$

$$=\int_0^{\pi} f(y)[\pi-g(y)]dy$$

$$=\pi\int_0^{\pi} f(y)dy-\int_0^{\pi} f(y)g(y)dy$$

$$=\pi\int_0^{\pi} f(x)dx-\int_0^{\pi} f(x)g(x)dx$$

$$\Rightarrow 2I=\pi\int_0^{\pi} f(x)dx \qquad \Rightarrow I=\frac{\pi}{2}\int_0^{\pi} f(x)dx$$

(β) $\int_0^{\pi}\frac{x\,\eta\mu x}{1+\sigma\upsilon\nu^2 x}dx$

$$f(x)=\frac{\eta\mu x}{1+\sigma\upsilon\nu^2 x}=\frac{\eta\mu(\pi-x)}{1+\sigma\upsilon\nu^2(\pi-x)}=f(\pi-x)$$

$g(x)=x$, $g(x)+g(\pi-x)=x+\pi-x=\pi$

$$\int_0^{\pi}\frac{x\,\eta\mu x}{1+\sigma\upsilon\nu^2 x}dx=\frac{\pi}{2}\int_0^{\pi}\frac{\eta\mu x}{1+\sigma\upsilon\nu^2 x}dx$$

$\sigma\upsilon\nu x=u \Rightarrow -\eta\mu x\,dx=du$

x	0	π
u	1	-1

$$=\frac{\pi}{2}\int_{-1}^{1}\frac{du}{1+u^2}=\left[\frac{\pi}{2}\tau o\xi\varepsilon\varphi u\right]_{-1}^{1}$$

$$=\frac{\pi}{2}\tau o\xi\varepsilon\varphi(1)-\frac{\pi}{2}\tau o\xi\varepsilon\varphi(-1)$$

$$=\frac{\pi^2}{8}+\frac{\pi^2}{8}=\frac{\pi^2}{4}$$

5.	(α) $\left.\begin{array}{l}A(\alpha\,\sigma\upsilon\nu\theta,\beta\,\eta\mu\theta)\\ B(\alpha\sigma\upsilon\nu\phi,\beta\,\eta\mu\phi)\end{array}\right\} \Rightarrow \left.\begin{array}{l}x_M=\dfrac{\alpha(\sigma\upsilon\nu\theta+\sigma\upsilon\nu\phi)}{2}\\ y_M=\dfrac{\beta(\eta\mu\theta+\eta\mu\phi)}{2}\end{array}\right\}$ $\lambda=\dfrac{\beta(\eta\mu\theta-\eta\mu\phi)}{\alpha(\sigma\upsilon\nu\theta-\sigma\upsilon\nu\phi)} \Rightarrow \dfrac{\eta\mu\theta-\eta\mu\phi}{\sigma\upsilon\nu\theta-\sigma\upsilon\nu\phi}=\dfrac{\alpha\cdot\lambda}{\beta}$ (1) $\left.\begin{array}{l}\eta\mu\theta+\eta\mu\phi=\dfrac{2y}{\beta}\\ \sigma\upsilon\nu\theta+\sigma\upsilon\nu\phi=\dfrac{2x}{\alpha}\end{array}\right\} \Rightarrow \dfrac{\eta\mu\theta+\eta\mu\phi}{\sigma\upsilon\nu\theta+\sigma\upsilon\nu\phi}=\dfrac{\alpha y}{\beta x}$ (2) (1) (2) $\Rightarrow \dfrac{\eta\mu^2\theta-\eta\mu^2\phi}{\sigma\upsilon\nu^2\theta-\sigma\upsilon\nu^2\phi}=\dfrac{\alpha\lambda}{\beta}\cdot\dfrac{\alpha y}{\beta x}=\dfrac{\lambda\alpha^2 y}{\beta^2 x}$ $\Rightarrow \dfrac{\beta^2}{\lambda\alpha^2}\left(\dfrac{\eta\mu^2\theta-\eta\mu^2\phi}{1-\eta\mu^2\theta-1+\eta\mu^2\phi}\right)x=y$ $\Rightarrow y=\dfrac{-\beta^2}{\lambda\alpha^2}x$ (β) $x=\pm\dfrac{\alpha^2\lambda}{\sqrt{\lambda^2\alpha^2+\beta^2}}$ $y=\mp\dfrac{\beta^2}{\sqrt{\lambda^2\alpha^2+\beta^2}}$ $\lambda_{\varepsilon\phi}=-\dfrac{\beta^2}{\alpha^2}\cdot\dfrac{x}{y}=-\dfrac{\beta^2}{\alpha^2}\left(\dfrac{\alpha^2\lambda}{-\beta^2}\right)=\lambda$	